Member's Default Utility Function for Default Fund Design Version 1 ("MDUF v1")

Technical Paper No.1: MDUF v1 Design

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1 Introduction

This paper provides greater technical detail into the design and the derivation of MDUF v1. It should be read following a review of the "Introduction Paper".

As discussed in the "Introduction Paper", providing retirement outcome solutions is a hugely challenging and complex area. Technically the retirement outcomes problem can be defined as:

"A dynamic, integrated consumption and investment decision problem."

This is a complex problem and there are two important elements to reflect on:

1. An integrated consumption and investment decision based on many factors such as members' characteristics and situation (acknowledging a heterogeneous population) and the large range of available products, solutions and services (yet shortage of product in some key areas). The word "integrated" is an important word to reflect on. At any point in time there is an optimal combined consumption and investment strategy. To determine one in isolation of the other will not result in the best possible solution.

2. A dynamic consumption and investment problem which should be reviewed regularly especially when market conditions and members' situation change.

This problem, combined with the difficulties experienced by the industry in communicating complex financial solutions to fund members who on average have low levels of financial literacy, creates an even greater challenge for the industry.

The industry lacks a clear objective. The proposed objective of superannuation set in the Superannuation (Objective) Bill 2016 is *"To provide income in retirement to substitute or supplement the Age Pension"*. However such an objective remains aspirational rather than precise. Without

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a precise definition it is not possible to undertake the quantitative work necessary to address the issue of retirement solution design.

While there are existing retirement outcome metrics they are generally flawed and fail to adequately represent the preferences of a default member. We created a working group of academics and industry professionals to determine an appropriate and sensible set of objectives for trustees to assume on behalf of their default fund members, members they know little about. We then converted these preferences into a mathematical function, known as a utility function. This function has been given the name "Member's Default Utility Function Version 1" or "MDUF v1" for short.

In this paper we provide greater detail, in a more complex setting compared to the "Introduction Paper", around the design of MDUF v1. We focus on describing how we derive the final form of MDUF v1 with a comprehensive set of discussions around the design from different aspects. We believe the final form of MDUF v1 is a credible and powerful metric that can be used to address the complex problem outlined above. For the solution techniques including a static solution and optimal dynamic solution, please refer to "Technical Papers No.2: Static Model" and "Technical Papers No.3: Optimal Dynamic Strategies".

2 What is a Utility Function?

Utility functions are mathematical functions that are designed to reflect the preferences of an individual. They can reduce the dimension of problems to something that is easier to measure and compare through ranking alternatives according to their utility outcomes. They can be very useful in objectively addressing important questions such as comparing investment opportunities, constructing portfolios, designing and comparing products and assessing the value of advice. A higher utility outcome is always preferred over a lower utility outcome. In theory, economically rational investors make choices consistent with maximising the expected value of the utility function when considering a range of possible utility outcomes accounting for risk factors such as investment returns and mortality experience. Utility can be defined over either wealth or consumption over time. However it can also be defined over a bundle of items. The use of a utility function involves a number of consumers' behavioural assumptions such as their risk preference, their reaction to changes in economic factors and their wealth level.

In a perfect world we would construct individual retirement plans for each member with a perfect understanding of their preferences. There are two significant hurdles we face in practice. The first one is the operational restrictions and costs which mean that members are commonly aggregated into pooled solutions; we therefore need to make broad assumptions of an 'average' member which might not truly reflect individual preferences. The second one is that academic literature suggests that even if the administrative restrictions did not exist, it is still very hard to accurately elicit the preferences of individuals. One example could be the basic processes for risk profiling applied by some parts of the financial planning industry. General questionnaires on risk preferences are put forward to clients but these results do not convert well into an economic measure of risk aversion.

Utility functions can take many forms. Widely used utility functions take mathematical forms such as power functions, log functions, linear functions, etc. with respect to wealth or consumption. For a default fund we seek a sensible paternal utility function appropriate for a trustee to assume on behalf of its membership. This could then be considered by all super funds in the design of their default funds. However they could 'opt-out' and develop their own version of a utility function if they believe they have greater insight into the preferences of the members in their fund. We suspect two types of 'opt-out' from other super funds:

- Homogeneous opt-out: a super fund believes their members possess very different characteristics in risk aversion, bequest etc. compared to those presented in MDUF v1.
- Heterogeneous opt-out: a super fund believes the heterogeneity of their members' characteristics cannot be represented by one single utility function. This would be the case if all members received financial advice and preference assessments.

In both cases, the concept and framework of utility maximisation remains highly relevant. Once established, a utility function becomes our objective beacon for guiding our forward looking decisions around the design of retirement solutions. It can be an excellent ex-ante guide for us to recommend retirement strategies over time. There are a number of aspects to be considered in designing a utility function:

- The utility function is designed from the trustee's perspective and as a result it reflects existing regulations and the spirit of proposed regulations.
- We assume no unique insights into members beyond their age, gender, balance and contribution levels.
- A trustee is obliged to make (sometimes paternal in nature) decisions that they believe is in the best interests of members. One output of this is default fund design. A member always has the right to opt out of the default.
- The utility function needs to achieve its purpose with minimal complexity and it should be broadly understandable by an appropriately well resourced super fund, possibly with guidance from a consultant.
- Highly debatable assumptions used in constructing the utility function need to be avoided.

Given all of this we name the function the "Member's Default Utility Function v1" (MDUF v1). The v1 acknowledges that we believe this function should be reviewed periodically as the industry continues to evolve, particularly as new research into members' preferences become available.

3 Derivation of the MDUF v1

This section details the steps we went through to derive the recommended form of MDUF v1. In the design of the function, we only take into account members' basic information collected by a super fund such as age, gender and balance.

3.1 Step 1 of MDUF v1

The step 1 form of the MDUF v1 is as follow:

$$U_0 = \sum_{t=0}^{T} \beta^t \, \frac{c_t^{1-\rho}}{1-\rho}, \tag{3.1}$$

where t is the time since members enter into retirement. c_t is the consumption amount at time t, $\rho > 0$ is the level of risk aversion (higher ρ means a person has greater risk aversion), β is the subjective utility discount factor that captures the retiree's time preference for near versus far-dated consumption, and T is a predetermined death time of the members. Function 3.1 shows how the utility values of a stream of individual consumptions can be summed up to generate total lifetime utility.

In the following we discuss key aspects of the step 1 function and explain how we get to this form of the function.

3.1.1 Using Discounted Utility

There is no evidence of individuals undertaking complex modelling and determining an integrated investment and consumption decision which maximises their expected discounted utility (DU). Nonetheless, we believe this framework is appropriate to serve as the basis of a sensible objective for trustees to assume on behalf of members on which they have little insight.

The discounted utility framework (though note that our parameterisation presented in step 1 involves no discounting) is not without challenge in the academic literature, notably Frederick et al. (2002). Some of the key concerns raised in the paper are consumption independence and stationary instantaneous utility. Consumption independence means no consideration of the impact of consumption changing year-to-year and stationary instantaneous utility is the assumption that the utility function and parameters like risk aversion remain constant through life. These concerns are valid but do not dissuade us from continuing on down the discounted utility path for the following reasons:

- The DU approach embodies many sensible messages: the lifetime experience of retirement income is important, and, via the use of concave utility curves, a drop in income results in a larger loss of utility than the extra utility experienced from the same quantum increase in income. If returns were certain and we knew when we would die then a DU approach would recommend smoothed income.
- The DU approach inherently focuses on sustainable outcomes, an approach that we believe is consistent with the responsibility of trustees. This is discussed in further detail in Section 3.1.4.
- There is a substantial amount of academic literature focusing on optimal consumption and investment decisions which have used DU as the representation of objectives.
- Most extensions or alternatives to the DU approach introduce components which significantly impact the tractability of the model and make it much harder to understand and be used by industry. Basically every additional variable introduced adds a dimension to the problem (the "curse of dimensionality" as it is known!).

Overall the DU framework is concluded to be the most appropriate approach for this project. It carries good high level messages, focuses on sustainable solutions, is relatively easy to understand, and is frequently used in academic research in optimal consumption and investment problems. In combination this provides a highly defensible base for trustees.

3.1.2 Lifetime Utility

In the history of Australia's defined contribution system, the industry focus has generally been on terminal wealth, namely the lump sum benefit available at retirement. However, the definition of superannuation and the consideration of managing longevity risk guides us to focus on retirement income. As a result, we want to discuss lifetime consumption problems in addition to terminal wealth.

Compared to the one-period utility functions which only focus on terminal wealth, ones that account for lifetime consumption are more complicated as they take into account streams of consumption over time.

For time additive utility function (a special case of time-separable utility function in general), the Lifetime utility is just the summation of the utility score of each consumption cash flow as shown in Figure 3.1.

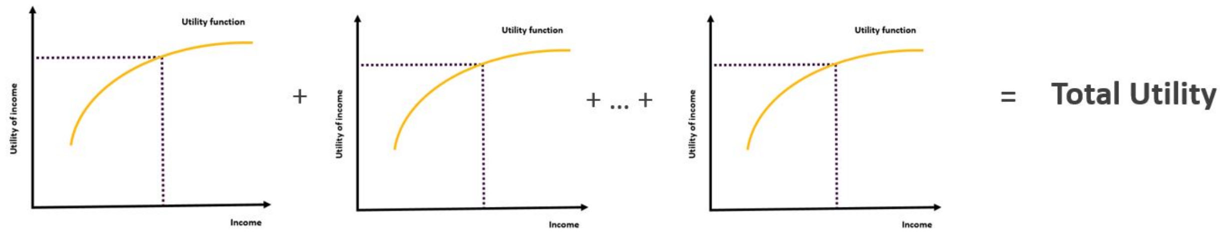


Figure 3.1: Lifetime utility calculation

3.1.3 Risk Aversion

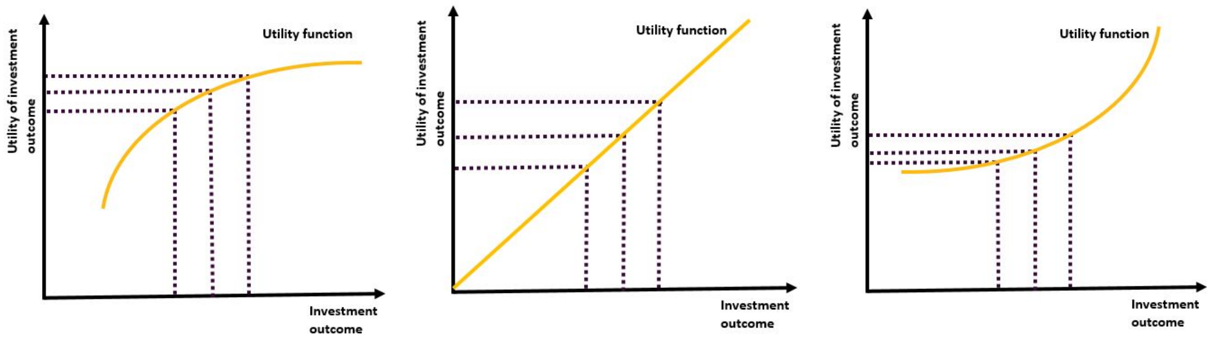
An individual can be identified as risk averse, risk neutral or risk loving depending on the shape of their utility curve. Table 3.1 explains what each of them means in detail.

Table 3.1: Comparing types of risk preference of investors

Risk averse	Risk neutral	Risk loving
An investor prefers a low level of risk on his investments. They will accept a higher level of risk only if the increase in expected returns is sufficient.	An investor is indifferent between choices with equal expected payoffs regardless of the riskiness of each choice	An investor seeks higher risk investments. They will only accept lower risk if it is accompanied by a sufficient improvement in expected returns.
Concave utility curve	Linear utility curve	Convex utility curve
Marginal utility is decreasing as wealth/income is growing.	Marginal utility is unchanged as wealth/income is growing.	Marginal utility is increasing as wealth/income is growing.
Marginal utility is lower for higher outcome.	Marginal utility is constant for all outcome levels.	Marginal utility is higher for higher outcome.

Over the long term, lower expected returns are usually associated with lower risk investments and higher expected returns are associated with higher risk investments. As most investors expect to be compensated for taking on additional risk, they are considered to be risk averse investors.

Risk aversion captures an individual's conservativeness towards risk. The more risk averse an individual is, the more pronounced is the utility loss resulting from a reduction in consumption than the utility gain from an increase in consumption of the same amount, i.e. the higher the risk aversion parameter, people are more afraid of loss.



(a) Risk averse (b) Risk neutral (c) Risk loving

Figure 3.2: Types of risk preference of investors

3.1.4 Constant Relative Risk Aversion (CRRA)

There are a lot of different classes of utility functions for risk averse investors. We decided to use the Constant Relative Risk Aversion (CRRA) utility function. Before we explain why we chose CRRA, it is important to introduce the concepts of absolute and relative risk aversion.

- **Absolute risk aversion** (ARA) measures the individual's risk aversion level as the absolute dollar value of their consumption level changes. Let $ARA(c)$ denote the absolute risk aversion at consumption level c. It is defined as the negative of the ratio between the second derivatives and the first derivatives of the utility function with respect to consumption as shown below:

$$ARA(c) = -\frac{u''(c)}{u'(c)}, \tag{3.2}$$

  where c denotes consumption, $u(c)$ denotes a utility function with respect to c, $u'(c)$ is the first order derivative of $u(c)$, and $u''(c)$ is the second order derivative of $u(c)$.

- **Relative risk aversion** (RRA) measures the individual's risk aversion level with respect to percentage changes in their consumption level. Let $RRA(c)$ denote the relative risk aversion at consumption level c. It is defined as the product of the absolute risk aversion value and the investors' consumption level, i.e.:

$$RRA(c) = c \times ARA(c) = -c \times \frac{u''(c)}{u'(c)}. \tag{3.3}$$

Table 3.2 explains the different types of risk aversions by considering the different attitudes of investors towards investing in risky assets when their consumption level changes. Absolute risk aversion focuses on dollar amount changes in risky investment and relative risk aversion focuses on the percentage changes of the portfolio invested in risky assets.

Table 3.2: Different types of risk aversions

	Increasing (I)	Constant (C)	Decreasing (D)
Absolute risk aversion (ARA)	IARA: the investor tends to increase the dollar amount invested in risky assets when his/her consumption level increases	CARA: the investor tends not to change the dollar amount invested in risky assets when his/her consumption level increases	DARA: the investor tends to decrease the dollar amount invested in risky assets when his/her consumption level increases
Relative risk aversion (RRA)	IRRA: the investor tends to increase the percentage of his/her wealth invested in risky assets when his/her consumption level increases	CRRA: the investor tends not to change the percentage of his/her wealth invested in risky assets when his/her consumption level increases	DRRA: the investor tends to decrease the percentage of his/her wealth invested in risky assets when his/her consumption level increases

For investors who are CARA, as their consumption level increases, the dollar amount they invest in risky assets would not change. For investors who are CRRA, as their consumption level increases, the percentage of their wealth invested in risky assets does not change.

Utility functions with constant risk aversion, i.e. CARA or CRRA, are the most commonly used in academic literature on maximization of expected utility of terminal wealth in DC accumulation phase. Some papers that discuss about using CARA utility function include Devolder et al. (2003) and Battocchio and Menoncin (2004). There are a lot of papers which discuss CRRA as the most widely used utility function especially when it is defined over life-cycle consumption. These include Tobin and Dolde (1971), Mehra and Prescott (1985), Gourinchas and Parker (2002), Chetty (2006) , Schechter (2007), Yogo (2009), Ameriks et al. (2011), and Lockwood (2014). Given our focus on income in retirement rather than lump sum, we chose CRRA over CARA for the design of MDUF v1.

In a simple one-period case, CRRA utility function can be expressed in the following equation:

$$u(c) = \begin{cases} \frac{c^{1-\rho}}{1-\rho} & \text{if } \rho > 0,\ \rho \neq 1 \text{ (This is known as power utility)} \\ \ln c & \text{if } \rho = 1 \text{ (This is known as log utility, a special case of power utility)} \end{cases} \tag{3.4}$$

where ρ is the constant relative risk aversion.[1] As explained earlier, individuals with a CRRA utility function will keep a constant proportion of their wealth invested in risky assets when their consumption level increases. The value of ρ determines the degree of risk aversion. A higher ρ means a higher degree of risk aversion as the marginal utility from having additional consumption is lower. We chose the power utility form over the log utility for MDUF v1. This is supported by Tobin and Dolde (1971), Schechter (2007), Yogo (2009), Ameriks et al. (2011), and Lockwood (2014) as they all suggested the value of ρ should be greater than 1.

Research has identified that individuals seek smooth income (see e.g., Hall, 1978; Modigliani and Brumberg, 1954). It is likely that this would result in lower income levels. We contend that an

[1] This can be confirmed using the definition of relative risk aversion, as shown in the following equation:

$$RRA(c) = -c \times \frac{u''(c)}{u'(c)} = -c \times \frac{-\rho(1-\rho)c^{-1-\rho}}{(1-\rho)c^{-\rho}} = \rho.$$

approach that priorities cash flow smoothing comes at the risk of unsustainability. When looked at through the lens of a trustee we do not believe it appropriate to explicitly target income smoothing.

We are motivated to consider whether the discounted utility framework based on CRRA can reflect a preference for smooth income. If investment returns were certain and we knew our date of death then our DU / CRRA framework would indeed recommend as optimal a strategy with constant income. However what happens in the real world environment when investment returns and mortality outcomes are uncertain? The optimal strategy would suggest a level of income that is sustainable by spreading the expected consumption evenly on a basis which accounts for the distribution of investment returns and mortality outcomes and maximises total expected utility. As a result a change in wealth due to market movements results in a change in sustainable income. This is an important message for trustees: the risk that is taken at a fund level at any time will impact the sustainable level of income not just this year but for all future years. The DU / CRRA framework provides a strong focus on sustainable income in retirement. If we acknowledge the dynamic nature of the problem we need to constantly reassess the optimal consumption and investment strategy. Any prioritisation of income smoothing is to the detriment of a sustainable retirement outcome strategy.

The coefficient of relative risk aversion represents the trade-off between a higher level of expected income and year-on-year variability in income. A higher coefficient of relative risk aversion implies a preference for smoother income at the cost of a lower level of expected income.

Overall we believe that the DU / CRRA framework is appropriate for MDUF v1. The core focus of DU / CRRA is sustainability of retirement income. The choice of the coefficient of relative risk aversion allows the consideration of the trade-off between high income preferences and smooth income preferences.

3.2 Step 2 of MDUF v1

In step 2 of the MDUF v1, we removed the subjective utility discount factor β and the form becomes:

$$U_0 = \sum_{t=0}^{T} \frac{c_t^{1-\rho}}{1-\rho}, \tag{3.5}$$

In the following we discuss why β was removed and explain some other considerations that led to this form of the function.

3.2.1 Subjective Utility Discount Factor

The total utility depends on how much the investors consume today at time 0 and on how much they consume in the future time $t = 1, 2, \ldots, T$. The parameter β is the subjective time preference discount factor given that we are comparing consumption in real terms. It captures the weight that investors place on the future relative to today. If $\beta = 1$, then they treat consumption today and in the future equally. Alternatively, if $\beta < 1$ then they value today's consumption more than the future.

The value of the subjective utility discount factor (i.e. β) has received a number of debates in the academic literature. Some argue that a rational individual should place equal value throughout life, so they believe that the subjective utility discount factor should be equal to 1 (see e.g., Broome, 1991; Elster, 1986; Rawls, 2009; Becker and Murphy, 1988). Looking from an individual's point of view, some philosophers (such as Zemach, 1987; Parfit, 1993) describe an individual as "a succession of overlapping selves related to varying degrees of memories" so they believe it is rational to discount future utility (Frederick, 1999). The values of subjective utility discount factor used in the literature are largely variable (Frederick, 1999). Through the lens of a trustee, it is appropriate to assume $\beta = 1$. This means we focus on a sustainable retirement income strategy through life rather than catering to potential myopic (short-sighted) biases which could place retirement outcomes at risk. In addition, from the view of trustees that represent many members of different cohorts, the intergenerational equity is an important issue, i.e. having a less than 1 utility discount factor would mean less value given to those who survive to older ages.

3.2.2 Elasticity of Intertemporal Substitution (EIS)

Elasticity of Intertemporal Substitution (EIS) considers more than time value of money, rather it focuses on real return of investments. Basically, it suggests that if market conditions change and investors form different perceptions of the future market return (real), then they will change their preference with regard to deferring consumption into the future. If they expect better future real return, then they might prefer to consume less today to leave a bit more money in the investment to grow for future consumption. The parameter of EIS determines the sensitivity of the investors' consumption behaviour towards changes in their expectations of the market real return. The EIS parameter of CRRA utility function is equal to the inverse of the risk aversion parameter ρ. This is a result of the choice of a time additive utility function

Historical research has shown the value of EIS to be relatively low. i.e. investors' consumption behaviour is insensitive towards changes in expectation for future market performance. In addition, it might be hard to incorporate EIS explicitly in practice because this would involve a much more complex multi-period utility function which is non-time-additive. This would make the whole model less tractable. As a result, we decided not to incorporate EIS explicitly here.

3.3 Step 3 of MDUF v1

In step 3 of the MDUF v1, we included the residual benefit component and the form becomes:

$$U_0 = \sum_{t=0}^{T} \frac{c_t^{1-\rho}}{1-\rho} + \frac{b_T^{1-\rho}}{1-\rho}\left(\frac{\phi}{1-\phi}\right)^{\rho} \tag{3.6}$$

where b_T is the level of wealth at time T which equals the amount of residual benefit if the person dies between $T-1$ and T, ϕ is the strength of residual benefit motive (higher ϕ means stronger residual benefit motive).

In the following we discuss why the residual benefit component was added.

3.3.1 Residual Benefit Motive

The residual benefit motive represents an individual's preference to leaving residual wealth at death to his/her beneficiary. Ceteris paribus, a higher residual benefit motive parameter represents a preference to leave more in the residual benefit.

Philosophically it is hard to place value on something that people will not experience because they are deceased. Some policymakers view superannuation as purely retirement income and place little value on what is leftover, yet people purchase life insurance for various reasons. There are also structural issues. The superannuation system has account-based or account value origins. Members think of super as their account balance. It will take many years, even decades, to change this focus. Under such a structure members and hence trustees would place value on the leftover amount when they pass away.

We believe that, when looked through the lens of the trustee, it is important to assign some value to any residual benefit.

1. There is a distinct risk of dying early in retirement. Assuming one were to retire today at age 65: then for a male (female) there is a 1.1% (0.6%) chance of dying in the first year of retirement and a 15.6% (9.9%) chance of dying in the first decade of retirement. In these cases we believe that it would be inappropriate for a trustee to design a post-retirement solution which places no value on any residual benefit;

2. The superannuation system is designed around the individual, not the household, yet over 65% of people retire with a partner. For households with a significant income difference between the two partners the residual account value provides the retirement outcome for the surviving (low income) partner;

3. Empirical research suggests that people do place value on the bequest aspect associated with a residual benefit. One example is Lockwood (2014), a research paper which attempts to model retirement solution design to align with the empirical evidence around bequest motive. It shows very consistent annuitization result from the model compared to the empirical data when allowing for relatively modest bequest motive.

4. Residual benefit acts as a reserve pool for many life events related to aged care, healthcare, travelling and family. This is an important point given we do not capture liquidity preference in MDUF v1.

Overall, we believe there is a very strong case for trustees to put some value on any residual benefit.

3.4 Step 4 of MDUF v1

In step 4 of the MDUF v1, we incorporated the uncertainty of outcomes due to investment risk. The form becomes:

$$U_0 = \mathbb{E}_0\left[\sum_{t=0}^{T}\left\{\frac{c_t^{1-\rho}}{1-\rho} + \frac{b_T^{1-\rho}}{1-\rho}\left(\frac{\phi}{1-\phi}\right)^{\rho}\right\}\right], \tag{3.7}$$

where $\mathbb{E}_0$ is the expectation operator with respect to time 0.

3.4.1 Investment Risk

In step 1 to 3, we ignore the fact that consumptions levels through time and residual benefit value can vary due to investment risk. We acknowledge that investment outcomes cannot be certain. Investment outcome in one period can in fact impact consumption not only in the next period but also all subsequent periods. As a result, expected utility needs to be calculated for the members' lifetime outcome; the expectation operator captures the distribution of all possible outcomes.

3.5 Step 5 of MDUF v1

In step 5 of the MDUF v1, we incorporate the uncertainty of outcomes due to mortality risk. The form becomes:

$$U_0 = \mathbb{E}_0\left[\sum_{t=0}^{T}\left\{ {}_tp_x \, \frac{c_t^{1-\rho}}{1-\rho} + {}_{t-1|}q_x \, \frac{b_t^{1-\rho}}{1-\rho}\left(\frac{\phi}{1-\phi}\right)^{\rho}\right\}\right], \tag{3.8}$$

where x is the inception age of a particular cohort, ${}_tp_x$ is the probability of being alive at age $x+t$ conditional on being alive at age x, ${}_{t-1|}q_x$ is the probability of dying between age $x+t-1$ and $x+t$ conditional on being alive at age x; T is no longer the predetermined death time of the members but the retirement planning horizon ($x+T$ is the maximum age).

The dynamics of wealth b_t is

$$\begin{aligned} b_1 &= (b_0 + P_0 + c_0)\left(1+\tilde{R}_1\right), \\ b_{t+1} &= (b_t + P_t - c_t)\left(1+\tilde{R}_{t+1}\right), \quad \text{for } t>0 \end{aligned} \tag{3.9}$$

where P_t is the amount of Age Pension entitlement received at time t if considered, and $\tilde{R}_t$ is the stochastic return p.a. of the investment portfolio from time $t-1$ to t.

In the following we discuss why mortality risk was incorporated.

3.5.1 Mortality Risk

In steps 1 to 4, we only considered known death time of members. We acknowledged that mortality outcomes cannot be certain (i.e. members do not know when they will die). Mortality outcomes impact the timing of any residual benefits as they are only paid out when the members die. Mortality outcomes also impact the size of consumption cash flows as the values will drop to zero for all subsequent periods following the death of the members. Given we can estimate the distribution of mortality, expected utility for the members' lifetime outcome can be calculated by directly incorporating the mortality probability in the function.

The expected lifetime utility can be calculated for different retirement strategies (e.g. different drawdown rules, different investment portfolios, dynamic or static strategies, etc.) and/or having various retirement products (e.g. life annuities). These utility scores can then be used to compare across retirement strategies to seek better retirement outcome for retirees.

3.6 Step 6 of MDUF v1

In step 6 of the MDUF v1 which is the final step, we recommend the parametrisation of the residual benefit motive strength parameter ϕ and the risk aversion level parameter ρ through extensive literature reviews and model calibrations.

3.6.1 Residual Benefit Motive Strength Parameter

The residual benefit motive strength parameter ϕ is chosen to be 0.83. This means the propensity to consume, which is calculated as the ratio of annual consumption to residual benefit is equal to $1 - \phi$ (17%). A 17% propensity to consume means if a retiree has 100 dollars, he/she will tend to spend 17% and save 83% in their accounts.

This choice of parameter is based on Lockwood (2014). We incorporate this parameter into our framework and justify the appropriateness of using this parameter in MDUF v1 through a simple model illustration. The detail of the illustration can be found in Appendix 7.1. The illustration shows that the ratio of total consumption to residual benefit value changes with respect to different residual benefit motive strength and the length of planning horizon. The key message is that it allows MDUF v1 to place a lower value of the residual benefit than the long-term income stream that it could generate. This prevents the development of bequest prioritisation strategies - "live on a low level of income and maximise the residual benefit". This is an important consideration of MDUF v1. As a result, we believe that the choice of $\phi = 0.83$ is appropriate to reflect our members' preference around residual benefit.

3.6.2 Risk Aversion Parameter

The risk aversion parameter ρ is chosen to be 8. This choice of parameter is a combined result of literature reviews and model calibration. We have reviewed a range of relevant academic literature in the field of CRRA utility defined over life-cycle consumption and bequest. A summary of the risk aversion parameters ρ used in a wide range of literature can be found in Table 7.1 in Appendix 7.2. Our finding is that the choice of ρ in most studies fall within the range of 1 to 10.

To further consider the appropriate parameter for MDUF v1, we developed and calibrated a lifecycle model using the assumptions in Table 3.3 to show the resulting year-to-year changes in retirement income based on $\rho = 2, 5, 8, 10$.

Table 3.3: Parameter values for base-case analysis. Sources are cited in brackets.

Parameter	**Explanation**	**Value**	**Source**
r_f	Risk-free rate	0.00%	Assumption
μ_R	Mean equity return	5.00%	Assumption
σ_R	Standard Deviation of equity return	15.00%	Assumption
ρ	Risk aversion	8	MDUF v1 Specification
ϕ	Residual benefit motive strength	0.83	Lockwood (2014)
b_0	Initial wealth (\$1,000)	500	Assumption
P_t	Age Pension Entitlement (\$1,000)	0	Assumption

We believe that the potential for year-to-year changes in retirement income of more than 10% would not be palatable to super fund members. Figure 3.3 shows the mean and 95% confidence intervals of the year-to-year percentage changes in consumption obtained in the MDUF v1 for different risk aversion parameter values. We observe that a risk aversion parameter of 8 delivers fairly reasonable variability in year-to-year consumption changes consistent with our view. This level is also within the range (1 to 10) used in many academic studies (for instance, Ameriks et al., 2011; Yogo, 2009; Mehra and Prescott, 1985; Friend and Blume, 1975).

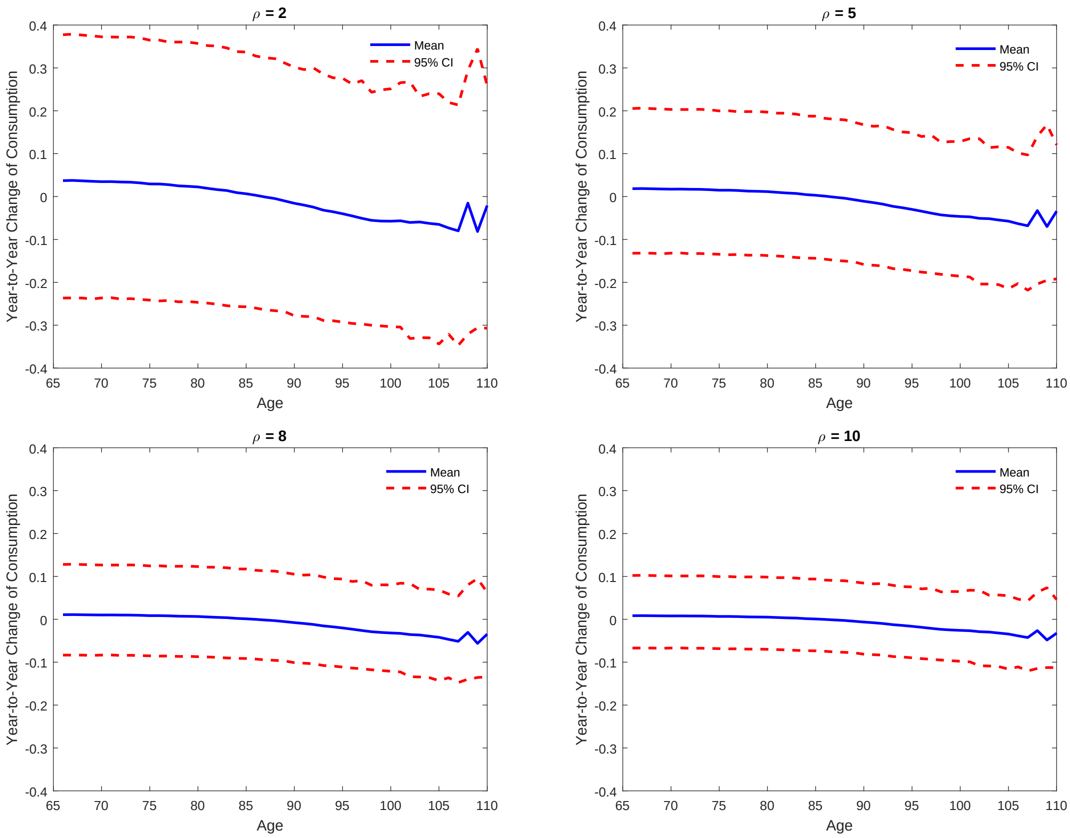


Figure 3.3: Consumption year-to-year percentage changes for different risk aversion parameters.

3.7 Recommended Form of MDUF v1

$$U_0 = \mathbb{E}_0 \left[\sum_{t=0}^{T} \left\{ {}_tp_x \frac{c_t^{1-\rho}}{1-\rho} + {}_{t-1|}q_x \frac{b_t^{1-\rho}}{1-\rho} \left(\frac{\phi}{1-\phi} \right)^{\rho} \right\} \right]$$

- x: the inception age of a particular cohort,
- T: the retirement planning horizon ($x + T$ is the maximum age),
- c_t: consumption in year t,
- b_t: level of wealth at time t which equals the amount of residual benefit if the person dies between $t - 1$ and t,
- ${}_tp_x$: the probability of being alive at age $x+t$ conditional on being alive at age x (Mortality rates can be sourced from the Australian Life Tables 2010-12 by Australian Government Actuary[2]),
- ${}_{t-1|}q_x$: the probability of dying between age $x + t - 1$ and $x + t$ conditional on being alive at age x (${}_{t-1|}q_x$ can be calculated from ${}_tp_x$),
- $\rho = 8$: level of risk aversion, and
- $\phi = 0.83$: strength of residual benefit motive.

[2]The Australian Life Tables 2010-12 can be downloaded via `http://www.aga.gov.au/publications/life_table_2010-12/`

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4 Other Measures Related to Utility

Utility can be used as a scoreboard of an individual's lifetime welfare, which is a ranking instead of an absolute score. It is difficult to perform quantitative comparisons directly based on utility scores.

There are indeed other measures related to utility which are widely used to investigate welfare gains or losses of different retirement strategies. This section describes key measures related to utility.

4.1 Certainty Equivalent Consumption (CEC)

Certainty Equivalent Consumption is calculated as the consumption level in the one-period utility function, i.e. equating $\frac{c_t^{1-\rho}}{1-\rho}$ to the lifetime utility calculated for the strategy using Equation 3.8. CEC, in essence, is a monotonic transformation of the lifetime utility. A higher level of the lifetime utility also corresponds to a higher CEC level. Note that CEC does not necessarily convey information of the actual level of consumption.

4.2 Wealth Gap (WG)

For a pair of cases, the wealth gap is calculated as the additional amount in the initial wealth in one case that can result in the same level of lifetime utility in the other case. We can compare cases with and without a certain retirement product (e.g. life annuity) in order to measure the dollar amount of welfare gains from having access to this product and using it optimally.

4.3 Extra Annual Return (EAR)

For a pair of cases, the extra annual return is calculated as the additional annual return of the fund investment performance in one case that can result in the higher level of life-time utility in the other case. We can compare cases with and without a certain retirement product (e.g. life annuity) in order to measure the welfare gains of having access to this product, in terms of annual investment returns.

5 What is Captured in MDUF v1?

Guiding principles of the MDUF v1 were:

1. Utility function is an appropriate tool to reflect our members' preference.

2. MDUF v1 should be a sensible representation of the retirement income problem, viewed through the lens of a trustee.

3. The focus is on lifetime consumption (income) rather than terminal wealth.

4. End-of-life residual benefit value is considered to recognise the value placed by members.
5. Rational behaviour of our members are assumed. Behavioural biases are not part of the consideration.
6. Where possible we stay close to the mainstream academic literature.
7. The framework chooses a simpler approach where possible to increase the ability for the industry to make use of MDUF v1.
8. The framework is tractable so it can be used for fund design.

Specific considerations regarding members's preferences, incorporated into the design of MDUF v1 include:

1. Higher income through life
2. Smooth income over time
3. Outliving savings is bad outcome
4. Residual benefit has value
5. People are risk averse

MDUF v1 does not capture liquidity preferences. There is currently no dominant stream of research on how to incorporate liquidity preferences into a preference function. This would be a valuable consideration in a version 2 project. In the meantime we would advise that super funds consider incorporating formal liquidity limits (modelled through life) into their retirement solution design.

It would be desirable for the MDUF v1 to perform a "straw man" role for super funds trying to determine appropriate objectives for their default members. In this respect the MDUF v1 would represent a proposal which a super fund could argue against in justifying their own set of objectives. MDUF v1 can be used in many ways including the following:

1. To compare a number of solutions by ranking them based on their utility scores.
2. To seek optimal solution within a single product. For example, by recommending an optimal consumption path and asset allocation inside an account-based pension.
3. To seek optimal solution across multiple products. For example, by recommending an optimal mix of products.
4. To quantify the value of advice by calculating the utility gain of better retirement strategies.
5. To quantify the utility cost of suboptimal solutions.
6. To help super funds with project prioritisation.

6 Conclusion

We believe that the recommended MDUF v1 is a credible and powerful metric that can be used to address the complex retirement outcome problem. This paper provides technical details of different design aspects of MDUF v1.

The life-cycle model proposed in this paper is sophisticated and flexible. After determining a final set of parameter values, we can apply the MDUF v1 for better retirement fund design. The MDUF v1 is not perfect and there are some aspects that the function can not address, particularly liquidity preferences. We will need to understand these limitations when we apply MDUF v1. As a result, there are cases when a trustee may want to step away from the "straw man" provided in the form of MDUF v1.

Specific reasons for a trustee to step away from MDUF v1 include:

1. may want to allow for behavioural biases.

2. may have greater member insight.

The trustees that wish to account for these aspects can choose to step outside the MDUF v1 through a number of ways:

1. Changing parameter values. For example, the trustee can choose a different risk aversion parameter value ρ based on their better understanding of their members. The trustee can also turn off residual benefit motive by setting ϕ equal to 0 if they believe their members do not value residual benefit at all.

2. Incorporating additional features into the function, such as a consumption floor, luxury bequest etc.

7 Appendix

7.1 Residual Benefit Motive Illustration

We illustrate the impact of residual benefit motive on consumption using an N-period simple example where individuals know when they will die and plan to consume a fixed level of consumption throughout their life.

The asset growth rate is assumed to be 0 in this simple example. The utility maximisation objective function can be written as:

$$\begin{aligned} & \max_{\{c\}} \left\{ N\ u(c) + v(b) \right\} \\ = & \max_{\{c\}} \left\{ N \frac{c^{1-\rho}}{1-\rho} + \left(\frac{\phi}{1-\phi} \right)^{\rho} \frac{b^{1-\rho}}{1-\rho} \right\}, \end{aligned} \tag{7.1}$$

where c is the streamlined consumption level and b is the residual benefit value at the predetermined end of horizon. Given an initial wealth of b_0, the wealth dynamics can be written as

$$b_0 = cN + b. \tag{7.2}$$

The optimal streamlined consumption is obtained by setting the first-order derivative of Equation (7.1) to 0, i.e.

$$\begin{aligned} 0 &= \frac{\partial N\ u(c)}{\partial c} + \frac{\partial v(b)}{\partial c} \\ &= \frac{N}{c^{\rho}} - N \left(\frac{\phi}{1-\phi} \right)^{\rho} \frac{1}{b^{\rho}}. \end{aligned} \tag{7.3}$$

By re-arranging the Equation 7.3, we obtain the following relationship between the total consumption vs. residual benefit value:

$$\frac{Nc}{b} = \frac{N(1-\phi)}{\phi}. \tag{7.4}$$

In our base case analysis where the residual benefit motive strength parameter, i.e. ϕ, is equal to 0.83. With a 20 years' horizon, the ratio of total consumption to residual benefit value is 4.10, implying that we are valuing much more on consumption than residual benefit value.

Figure 7.1 shows how the ratio of total consumption to residual benefit value changes with respect to different residual benefit motive strength levels (ϕ) and horizon lengths (N). We see that the ratio of total consumption to residual benefit value increases as the time horizon is longer and the residual benefit motive strength is lower. For a very extreme case where residual benefit motive strength of 1, which means that the individual only values the amount in the residual benefit, the ratio of total consumption to residual benefit value is constant at 0. For a residual benefit motive strength of around 0.7, we see that the ratio of total consumption to residual benefit value can be as high as 16 when the time horizon is 30 years. At a residual benefit motive strength of 0.5, the ratio of total consumption to residual benefit value can be as high as 30 if the individual's horizon is 30 years.

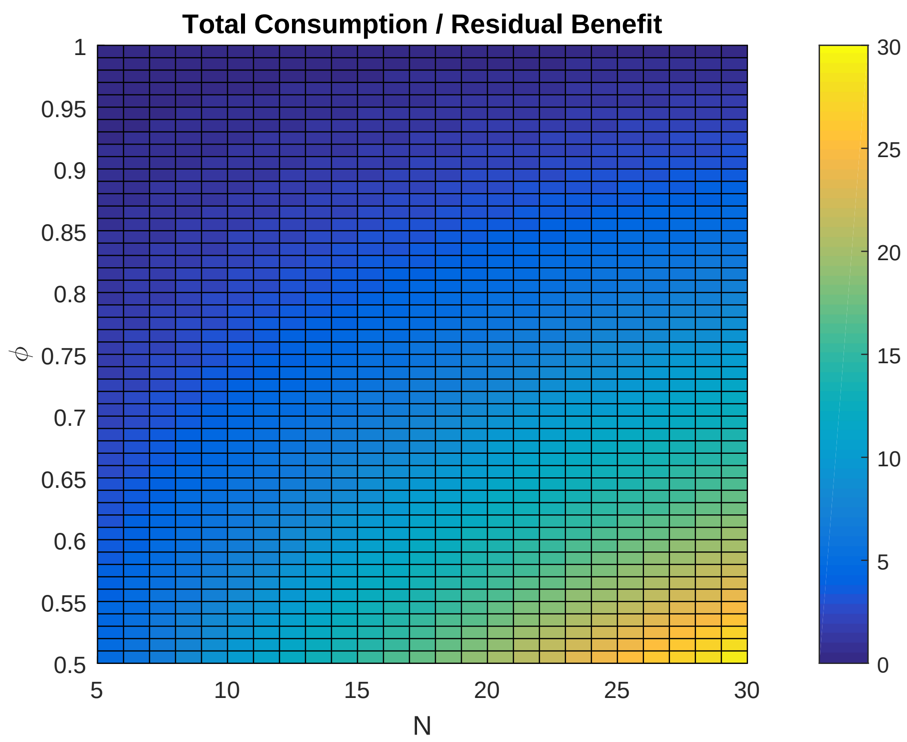

Figure 7.1: Ratio of total consumption to residual benefit value for different residual benefit motive and horizon lengths, without taking into account the Age Pension.

7.2 Choice of Risk Aversion Parameter

Paper	Utility Function	Parameter	Supporting evidence
Tobin and Dolde (1971)	CRRA defined over life-cycle consumption with borrowing constraint (liquidity constraint)	$\rho = 1.5$	Chose 1.5 to fit the observed life cycle savings patterns
Mehra and Prescott (1985)	CRRA defined over life-cycle consumption	$\rho < 10$	Concluded based on numerous studies in the past showing ρ is always estimated to be smaller than 10. This relies on the estimated 'price of risk' computed by Friend and Blume (1975) which is $\rho > 1$.
Gourinchas and Parker (2002)	CRRA defined over life-cycle consumption	$\rho < 2$	Calibrated by the life-cycle model using Consumer Expenditure Survey data
Chetty (2006)	CRRA defined over life-cycle consumption and labour supply	$\rho < 2$	Calibrated by the life-cycle model using data on labour supply behaviour

Schechter (2007)	CRRA defined over life-cycle consumption. Distinct between risk aversion for consumption and wealth. Meyer and Meyer (2005) show that ρ for consumption is about 5 times higher than the one for wealth ρ	$\rho = 1.9$	Calculated using survey data on income and experimental data on bet choice in a risk game for rural Paraguayan households and assuming individuals can not save. Can not save means income and consumption are identical.
		$\rho > 1,000$	Same method but assuming individuals can save. This gives empirical evidence for narrow bracketing
Yogo (2009)	CRRA defined over life-cycle consumption	$\rho = 5$	Calibrated by the life-cycle model based on the Health and Retirement Study
Ameriks et al. (2011)	CRRA defined over life-cycle consumption and end-of-life utility from bequsets	$\rho = 2, 3, 5, 10$	Selected based on other literatures: $\rho < 2$ (Gourinchas and Parker, 2002), $\rho > 3$ for a lot of asset pricing studies.
Lockwood (2014)	CRRA defined over life-cycle consumption and bequests	$\rho = 2.5$	Calibrated by life-cycle model to match retiree's savings including bequest motives and long-term care insurance choice. Model results range from 2.00 to 2.62.
Janecek (2002)	CRRA defined based on personal investment wealth	$\rho = 30$	For average investor based on empirical evidence from a number of experimental results.
		$\rho = 20$	For investors with enough experience based on empirical evidence.
		$\rho > 300$	For investors with little risk-taking experience and distaste for risk endeavors based on empirical evidence.

Table 7.1: Summary of Risk aversion parameter considered in academic literature

References

Ameriks, J., Caplin, A., Laufer, S. and Van Nieuwerburgh, S. (2011), 'The joy of giving or assisted living? using strategic surveys to separate public care aversion from bequest motives', *Journal of Finance* **66**(2), 519–561.

Battocchio, P. and Menoncin, F. (2004), 'Optimal pension management in a stochastic framework', *Insurance: Mathematics and Economics* **34**(1), 79–95.

Becker, G. S. and Murphy, K. M. (1988), 'A theory of rational addiction', *Journal of Political Economy* pp. 675–700.

Broome, J. (1991), 'Weighing goods: Equality, uncertainty and time'.

Chetty, R. (2006), 'A new method of estimating risk aversion', *The American Economic Review* pp. 1821–1834.

Devolder, P., Princep, M. B. and Fabian, I. D. (2003), 'Stochastic optimal control of annuity contracts', *Insurance: Mathematics and Economics* **33**(2), 227–238.

Elster, J. (1986), *Rational choice*, NYU Press.

Frederick, S. (1999), Discounting, time preference, and identity, PhD thesis, Carnegie Mellon University.

Frederick, S., Loewenstein, G. and O'donoghue, T. (2002), 'Time discounting and time preference: A critical review', *Journal of Economic Literature* **40**(2), 351–401.

Friend, I. and Blume, M. E. (1975), 'The demand for risky assets', *The American Economic Review* pp. 900–922.

Gourinchas, P.-O. and Parker, J. A. (2002), 'Consumption over the life cycle', *Econometrica* **70**(1), 47–89.

Hall, R. E. (1978), 'Stochastic implications of the life cycle-permanent income hypothesis: Theory and evidence', *Journal of Political Economy* **86**(6), 971–987.

Janecek, K. (2002), What is a realistic aversion to risk for real-world individual investors, Technical report, Citeseer.

Lockwood, L. M. (2014), Incidental bequests: Bequest motives and the choice to self-insure late-life risks, Technical report, National Bureau of Economic Research.

Mehra, R. and Prescott, E. C. (1985), 'The equity premium: A puzzle', *Journal of Monetary Economics* **15**(2), 145–161.

Meyer, D. J. and Meyer, J. (2005), 'Relative risk aversion: What do we know?', *Journal of Risk and Uncertainty* **31**(3), 243–262.

Modigliani, F. and Brumberg, R. (1954), 'Utility analysis and the consumption function: An interpretation of cross-section data', *Post-Keynesian Economics* **1**.

Parfit, D. (1993), 'The indeterminacy of identity: A reply to brueckner', *Philosophical Studies* **70**(1), 23–33.

Rawls, J. (2009), *A theory of justice*, Harvard university press.

Schechter, L. (2007), 'Risk aversion and expected-utility theory: A calibration exercise', *Journal of Risk and Uncertainty* **35**(1), 67–76.

Tobin, J. and Dolde, W. (1971), Wealth, liquidity and consumption, *in* 'Consumer spending and monetary policy: The linkages', Vol. 5, Federal Reserve Bank of Boston Boston, pp. 99–146.

Yogo, M. (2009), Portfolio choice in retirement: Health risk and the demand for annuities, housing, and risky assets, Technical report, National Bureau of Economic Research.

Zemach, E. M. (1987), 'Looking out for number one', *Philosophy and Phenomenological Research* **48**(2), 209–233.